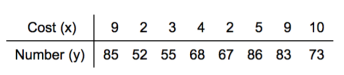
Predict 400 Fall 2016 Midterm – Zeeshan Latifi

1. The paired data below consist of the costs of advertising (in thousands of dollars) and the number of widgets sold (in thousands). Determine the least squares line and use this to predict the number of widgets sold if the cost of advertising is $9,000. Using Python, create a scatterplot and include the least squares line.



Python Code:

import numpy as np

import matplotlib.pyplot as plt

from scipy import stats

cost = np.array([9,2,3,4,2,5,9,10])

num\_widg = np.array([85,52,55,68,67,86,83,73])

plt.xlabel('Cost')

plt.ylabel('Number of Widgets')

plt.title ('Midterm #1')

plt.scatter(cost,num\_widg)

slope, intercept, r\_value, p\_value, std\_err = stats.linregress(cost,num\_widg)

print('The slope of the least squares line is')

print(slope)

print('The y-intercept of the least squares line is')

print(intercept)

y = slope\*cost +intercept

plt.plot(cost,y)

plt.show()

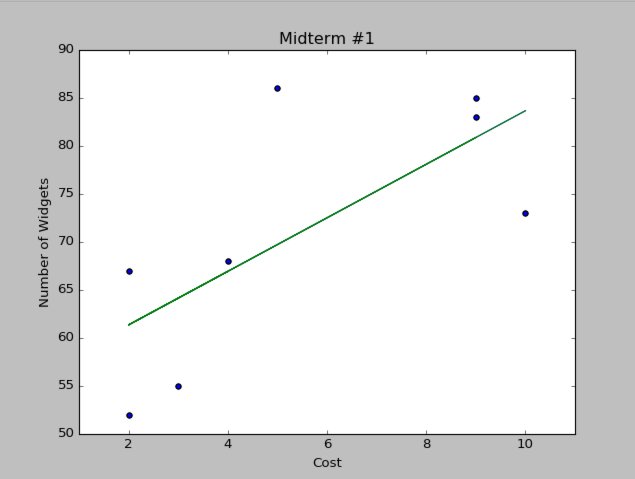
Output:

The slope of the least squares line is

2.78846153846

The y-intercept of the least squares line is

55.7884615385



Answer: The lease square line according to the points given is y = 2.789x + 55.789

1. Janet, Tim, and Josh work on an assembly line at a manufacturing company. The assembly of the company’s best-selling product can be completed if Janet and Tim work together for 4 hours and Josh works alone for 2 hours; or if Janet and Tim work together for 2 hours and Josh works alone for 5 hours; or if Janet works alone for 6 hours, Tim works alone for 2 hours, and Josh works alone for 1 hour. Due to budget constraints one employee must be laid off. Decide which employee should be laid off by determining the time it takes each employee working alone to assemble this product and selecting the least productive employee.

Janet = x

Tim = y

Josh = z

4x + 4y + 2z = 1

2x + 2y + 5z = 1

6x + 2y + z = 1

|  |  |  |  |
| --- | --- | --- | --- |
| 4 | 4 | 2 | 1 |
| 2 | 2 | 5 | 1 |
| 6 | 2 | 1 | 1 |

1/4 R1 -> R1

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 1 | 1/2 | 1/4 |
| 2 | 2 | 5 | 1 |
| 6 | 2 | 1 | 1 |

2R1 + (-R2) -> R2

2 2 1 ½

-2 -2 -5 -1

**0 0 -4 -1/2**

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 1 | 1/2 | 1/4 |
| 0 | 0 | -4 | -1/2 |
| 6 | 2 | 1 | 1 |

6R1 + (-R3) -> R3

6 6 3 3/2

-6 -2 -1 -1

**0 4 2 ½**

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 1 | 1/2 | 1/4 |
| 0 | 0 | -4 | -1/2 |
| 0 | 4 | 2 | 1/2 |

¼R3 -> R3 Swap R2 and R3

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 1 | 1/2 | 1/4 |
| 0 | 1 | ½ | 1/8 |
| 0 | 0 | -4 | -1/2 |

R1 + (-R2) -> R1

1 1 ½ ¼

0 -1 -1/2 -1/8

**1 0 0 1/8**

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0 | 0 | 1/8 |
| 0 | 1 | ½ | 1/8 |
| 0 | 0 | -4 | -1/2 |

-1/4R3 -> R3

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0 | 0 | 1/8 |
| 0 | 1 | ½ | 1/8 |
| 0 | 0 | 1 | 1/8 |

½R3 -> R3

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0 | 0 | 1/8 |
| 0 | 1 | ½ | 1/8 |
| 0 | 0 | ½ | 1/16 |

R2 + (-R3) -> R2

0 1 ½ 1/8

0 0 -1/2 -1/16

**0 1 0 1/16**

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0 | 0 | 1/8 |
| 0 | 1 | 0 | 1/16 |
| 0 | 0 | ½ | 1/16 |

2R3 -> R3

|  |  |  |  |
| --- | --- | --- | --- |
| **1** | **0** | **0** | **1/8** |
| **0** | **1** | **0** | **1/16** |
| **0** | **0** | **1** | **1/8** |

Janet = x = 1/8 hour

Tim = y = 1/16 hour

Josh = z = 1/8 hour

Answer: Lay off Tim

1. A company makes 3 types of cable. Cable A requires 3 black wires, 3 white wires, and 2 red wires. Cable B requires 1 black, 2 white, and 1 red. Cable C requires 2 black, 1 white, and 2 red. If 95 black wires, 100 white wires, and 85 red wires were used, how many of each cable were made?

Cable A = 3b 3w 2r

Cable B = 1b 2w 1r

Cable C = 2b 1w 2r

3a + 1b + 2c = 95

3a + 2b + 1c = 100

2a + 1b + 2c = 85

|  |  |  |  |
| --- | --- | --- | --- |
| 3 | 1 | 2 | 95 |
| 3 | 2 | 1 | 100 |
| 2 | 1 | 2 | 85 |

1/3R1 -> R1

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 1/3 | 2/3 | 95/3 |
| 3 | 2 | 1 | 100 |
| 2 | 1 | 2 | 85 |

-3R1 + R2 -> R2

-3 -1 -2 -95

3 2 1 100

**0 1 -1 5**

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 1/3 | 2/3 | 95/3 |
| 0 | 1 | -1 | 5 |
| 2 | 1 | 2 | 85 |

-2R1 + R3 -> R3

-2 -2/3 -4/3 -190/3

2 1 2 85

**0 1/3 2/3 65/3**

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 1/3 | 2/3 | 95/3 |
| 0 | 1 | -1 | 5 |
| 0 | 1/3 | 2/3 | 65/3 |

R1+ (-R3) -> R1

1 1/3 2/3 95/3

0 -1/3 - 2/3 -65/3

**1 0 0 10**

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0 | 0 | 10 |
| 0 | 1 | -1 | 5 |
| 0 | 1/3 | 2/3 | 65/3 |

3R3 -> R3

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0 | 0 | 10 |
| 0 | 1 | -1 | 5 |
| 0 | 3 | 2 | 65 |

R2 + (-R3) -> R3

0 1 -1 5

0 -1 -2 -65

**0 0 -3 -60**

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0 | 0 | 10 |
| 0 | 1 | -1 | 5 |
| 0 | 0 | -3 | -60 |

3R2 + (-R3) -> R2

0 3 -3 15

0 0 3 60

**0 3 0 75**

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0 | 0 | 10 |
| 0 | 3 | 0 | 75 |
| 0 | 0 | -3 | -60 |

1/3R2 -> R2

-1/3R3 -> R3

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0 | 0 | 10 |
| 0 | 1 | 0 | 25 |
| 0 | 0 | 1 | 20 |

Cable A = 10 cables were made

Cable B = 25 cables were made

Cable C = 20 cables were made

1. Gary’s lawn supply store has 165kg of rye grass seed and 93kg of bluegrass seed. He plans to sell two different grass seed mixes. One mix will contain half rye grass seed and half bluegrass seed and will sell for $8.25 per kg. The other mix will contain 3⁄4 rye grass seed and 1⁄4 bluegrass seed and will sell for $7.50 per kg. Use Python to determine how many kilograms of each seed mixture he should prepare for the maximum revenue, and the maximum revenue.

Python Code:

import pulp

from pulp import \*

# declare variables

x1 = LpVariable("x1", 0, None) # x1>=0

x2 = LpVariable("x2", 0, None) # x2>=0

# problem statement

prob = LpProblem("problem", LpMaximize)

# defines the constraints

prob += 0.5\*x1 + 0.75\*x2 <= 165

prob += 0.5\*x1 + 0.25\*x2 <= 93

# objective function to maximize

prob += 8.25\*x1 + 7.50\*x2

# solve the problem

status = prob.solve()

LpStatus[status]

rev = 8.25\*value(x1) +7.50\*value(x2)

# print the results

print('kgs that are for 0.5 rye grass and 0.5 bluegrass mix is')

print(value(x1))

print('kgs that are for 0.75 rye grass and 0.25 bluegrass mix is')

print(value(x2))

print("Actual Revenue is")

print(rev)

Output:

kgs that are for 0.5 rye grass and 0.5 bluegrass mix is

114.0

kgs that are for 0.75 rye grass and 0.25 bluegrass mix is

144.0

Actual Revenue is

2020.5

1. Gary Johnson’s campaign wants to hire regional strategists and assistants to fill its staffing needs at a minimum cost. The average monthly salary of a strategist is $2400 and the average monthly salary of an assistant is $1100. The campaign can hire up to 35 staff members and needs at least 20 to run properly. They must have at least 10 assistants and may have up to 3 assistants for every 2 strategists. Using Python, graph the feasible region and determine the number of strategists and assistants the campaign should hire.

Python Code:

import matplotlib.pyplot

from matplotlib.pyplot import \*

import numpy

from numpy import \*

x=arange(0,100.1,0.1)

y=arange(0,100.1,0.1)

y1= -1\*x +35.0

y2= -1\*x +20.0

y3= (3.0/2.0)\*x

y4 = 0.0\*x +10

xlim(0,40)

ylim(0,40)

xlabel('Number of Strategists')

ylabel('Number of Assistants')

title('Midterm Number 5')

plot(x,y1,'b')

plot(x,y2,'r')

plot(x,y3,'y')

plot(x,y4,'g')

legend(['x+y <= 35','x+y >= 20', 'y = 3/2x','y >= 10'])

x= [8, 10, 25, 14]

y= [12, 10, 10, 21]

fill(x,y, color='grey', alpha=0.2)

show()

obj= matrix([2400.0,1100.0])

obj= transpose(obj)

corners= matrix([x,y])

corners= transpose(corners)

result= dot(corners,obj)

print ('Value of Objective Function at Each Corner Point:\n'), result

Output:

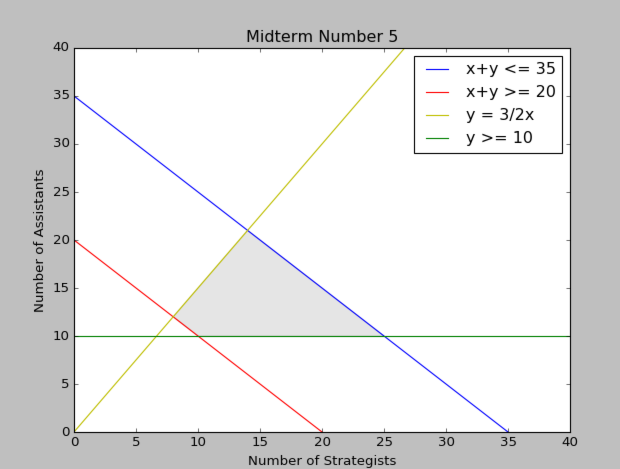
Value of Objective Function at Each Corner Point:

[[ 32400.]

[ 35000.]

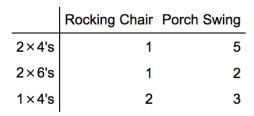
[ 71000.]

[ 56700.]]



Answer: The campaign should hire 8 strategists and 12 assistants, the total cost would be $32,400.

1. Honest Abe builds rocking chairs and porch swings and sells them at a local flea market. The lumber requirements are given below. Currently, Abe has 100 2 x 4’s, 55 2 x 6’s, 100 1 x 4’s in stock. Rocking chairs sell for $40 and porch swings sell for $85. How many pieces of lumber are left over after Abe has built the number of chairs and swings that maximize his revenue.



x1 = rocking chair

x2 = porch swing

z = 40x1 + 85x2

x1 + 5x2 <=100

x1+ 2x2 <= 55

2x1 + 3x2 <= 100

x1 x2 s1 s2 s3 z

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 1 | 5 | 1 | 0 | 0 | 0 | 100 |
| 1 | 2 | 0 | 1 | 0 | 0 | 55 |
| 2 | 3 | 0 | 0 | 1 | 0 | 100 |
| -40 | -85 | 0 | 0 | 0 | 1 | 0 |

1/5R1 -> R1

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 1/5 | 1 | 1/5 | 0 | 0 | 0 | 20 |
| 1 | 2 | 0 | 1 | 0 | 0 | 55 |
| 2 | 3 | 0 | 0 | 1 | 0 | 100 |
| -40 | -85 | 0 | 0 | 0 | 1 | 0 |

-2R1 + R2 -> R2

-2/5 -2 -2.5 0 0 0 -40

1 2 0 1 0 0 55

**3/5 0 -2/5 1 0 0 15**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 1/5 | 1 | 1/5 | 0 | 0 | 0 | 20 |
| 3/5 | 0 | -2/5 | 1 | 0 | 0 | 15 |
| 2 | 3 | 0 | 0 | 1 | 0 | 100 |
| -40 | -85 | 0 | 0 | 0 | 1 | 0 |

-3R1 + R3 -> R3

-3/5 -3 -3/5 0 0 0 -60

2 3 0 0 1 0 100

**7/5 0 -3/5 0 1 0 40**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 1/5 | 1 | 1/5 | 0 | 0 | 0 | 20 |
| 3/5 | 0 | -2/5 | 1 | 0 | 0 | 15 |
| 7/5 | 0 | -3/5 | 0 | 1 | 0 | 40 |
| -40 | -85 | 0 | 0 | 0 | 1 | 0 |

85R1 + R4 -> R4

17 85 17 0 0 0 1700

-40 -85 0 0 0 1 0

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 1/5 | 1 | 1/5 | 0 | 0 | 0 | 20 |
| 3/5 | 0 | -2/5 | 1 | 0 | 0 | 15 |
| 7/5 | 0 | -3/5 | 0 | 1 | 0 | 40 |
| -23 | 0 | 17 | 0 | 0 | 1 | 1700 |

5/3R2 -> R2

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 1/5 | 1 | 1/5 | 0 | 0 | 0 | 20 |
| 1 | 0 | -2/3 | 5/3 | 0 | 0 | 25 |
| 7/5 | 0 | -3/5 | 0 | 1 | 0 | 40 |
| -23 | 0 | 17 | 0 | 0 | 1 | 1700 |

5R1 + (-R2) -> R1

1 5 1 0 0 0 100

-1 0 2/3 -5/3 0 0 -25

**0 5 5/3 -5/3 0 0 75**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 0 | 5 | 5/3 | -5/3 | 0 | 0 | 75 |
| 1 | 0 | -2/3 | 5/3 | 0 | 0 | 25 |
| 7/5 | 0 | -3/5 | 0 | 1 | 0 | 40 |
| -23 | 0 | 17 | 0 | 0 | 1 | 1700 |

-7/5R2 + R3 -> R3

-7/5 0 14/15 -7/3 0 0 -35

7/5 0 -3/5 0 1 0 40

**0 0 1/3 -7/3 1 0 5**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 0 | 5 | 5/3 | -5/3 | 0 | 0 | 75 |
| 1 | 0 | -2/3 | 5/3 | 0 | 0 | 25 |
| 0 | 0 | 1/3 | -7/3 | 1 | 0 | 5 |
| -23 | 0 | 17 | 0 | 0 | 1 | 1700 |

23R2 + R4 -> R4

1/5R1 -> R1

23 0 -46/3 115/3 0 0 575

-23 0 17 0 0 0 1700

**0 0 5/3 115/3 0 1 2275**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 1/3 | -1/3 | 0 | 0 | 15 |
| 1 | 0 | -2/3 | 5/3 | 0 | 0 | 25 |
| 0 | 0 | 1/3 | -7/3 | 1 | 0 | 5 |
| 0 | 0 | 5/3 | 115/3 | 0 | 1 | 2275 |

Answer: Abe will need to build 25 rocking chairs and 15 porch swings to maximize his revenue. Abe will make $2,275 dollars of revenue.

25 + 5(15) =100

25 + 2(15) = 55

2(25) + 3(15) = 95

Abe will not have any leftover 2 x 4’s or 2 x 6’s. He will have 5 left over 1 x 4’s.

1. To properly stock her restaurant each week, Kaitlyn needs at least 10lbs of carrots, 12lbs of onions, and 20lbs of potatoes. One of her suppliers offers three packages containing combinations of these vegetables. Package 1 contains 4lbs of carrots and 3lbs of onions. Package 2 contains 1lb of carrots, 2lbs of onions, and 4lbs of potatoes. Package 3 contains 10 pounds of carrots, 1lb of onions, and 5lbs of potatoes. Package 1 costs $40, Package 2 costs $50, and package 3 costs $10. Using Python, determine how many of each package Kaitlyn should purchase each week to minimize her costs.

Package 1 (x1) -> 4 lb carrots 3 lb onions

Package 2 (x2) -> 1 lb carrots 2 lb onions 4 lbs potatoes

Package 3 (x3) -> 10 lb carrots 1 lb onions 5 lbs potatoes

P = 40x1 + 50x2 + 10x3

4x1 + 1x2 + 10x3 >= 10

3x1 + 2x2 + 1x3 >= 12

4x2 + 5x3 >= 20

Python Code:

import pulp

from pulp import \*

# declare your variables

x1 = LpVariable("x1", 0, None) # x1>=0

x2 = LpVariable("x2", 0, None) # x2>=0

x3 = LpVariable("x3", 0, None) # x3>=0

# defines the problem

prob = LpProblem("problem", LpMinimize)

# defines the constraints

prob += 4\*x1 + 1\*x2 + 10\*x3 >= 10

prob += 3\*x1 + 2\*x2 + 1\*x3 >= 12

prob += 4\*x2 + 5\*x3 >= 20

# defines the objective function to minimize

prob += 40\*x1 + 50\*x2 + 10\*x3

# solve the problem

status = prob.solve()

LpStatus[status]

min\_cost = 40\*(value(x1)) + 50\*(value(x2)) + 10\*(value(x3))

# print the results

print('The amount to order package 1 per week is')

print(value(x1))

print('The amount to order package 2 per week is')

print(value(x2))

print('The amount to order package 3 per week is')

print(value(x3))

print('The minimum cost in dollars is')

print(min\_cost)

Output:

The amount to order package 1 per week is

0.0

The amount to order package 2 per week is

0.0

The amount to order package 3 per week is

12.0

The minimum cost in dollars is

120.0

Answer:

Kaitlyn should purchase 12 packages of package 3 to minimize her costs. She should not purchase package 1 and 2.

1. A manufacturer makes three different drinks. Drink 1 contains 16% juice and 84% carbonated water. Drink 2 contains 50% juice and 50% carbonated water. Drink 3 contains 84% juice and 16% carbonated water. The manufacturer currently has available 100 liters of juice and 140 liters of carbonated water. Drink 1 sells for $1 per liter, Drink 2 sells for $2 per liter, and Drink 3 sells for $3.80 per liter. How many liters of each drink should the manufacturer make to maximize revenues?

Python Code:

import pulp

from pulp import \*

# declare your variables

x1 = LpVariable("x1", 0, None) # x1>=0

x2 = LpVariable("x2", 0, None) # x2>=0

x3 = LpVariable("x3", 0, None) # x3>=0

# defines the problem

prob = LpProblem("problem", LpMaximize)

# defines the constraints

prob += 0.16\*x1 + 0.5\*x2 + 0.84\*x3 <= 100

prob += 0.84\*x1 + 0.5\*x2 + 0.16\*x3 <= 140

# defines the objective function to maximize

prob += 1.0\*x1 + 2.0\*x2 + 3.80\*x3

# solve the problem

status = prob.solve()

LpStatus[status]

rev = 1.0\*value(x1) + 2.0\*value(x2) + 3.80\*(value(x3))

# print the results

print('Liters of Drink 1 to make in order to maximize revenue is')

print(value(x1))

print('Liters of Drink 2 to make in order to maximize revenue is')

print(value(x2))

print('Liters of Drink 3 to make in order to maximize revenue is')

print(value(x3))

print("Actual Revenue is")

print(rev)

Output:

Liters of Drink 1 to make in order to maximize revenue is

149.41176

Liters of Drink 2 to make in order to maximize revenue is

0.0

Liters of Drink 3 to make in order to maximize revenue is

90.588235

Actual Revenue is

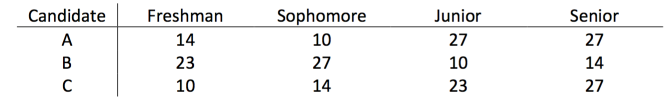
493.647053

Answer:

In order to maximize profits, the manufacturer should make 149 Liters of Drink 1, 0 Liters of Drink 2, and 90 Liters of Drink 3.

Actual Revenue would be $493.65

1. A random sample of Northwestern students was asked their top choice for Student Senate President and the data are in the table below. If a student is selected at random, find the probability that the student’s top choice is Candidate C given that the student is a junior or senior.



P(E|F) = P(E F)/P(F)

P(E F) = Juniors and Seniors that have voted for Candidate C = 23 + 27 = 50

P(F) = Juniors and Seniors that have voted 27 + 10 + 23 + 27 + 14 + 27 = 128

P(E|F) = 50/128 = 25/64 ~ 39.1%

1. The incidence of a certain disease in Evanston is 2%. A new test has been developed to diagnose the disease. Using this test, 95% of those who have the disease test positive while 7% of those who do not have the disease test positive (“false positive”). If a person tests positive, what is the probability that he or she actually has the disease?

E = Have the disease

F = Test indicates you have the disease

Probability of the population that has the disease = 0.02

Probability of the population that has the disease and the test indicates = 0.95

Probability of the population that does not have the disease = 0.98

Probability of the population that does not have the disease but the test indicates = .07

(0.02)(0.95) = 0.216895

(0.02)(0.95) + (0.98)(0.7)

Answer: The probability of someone that tests positive actually has the disease is about 21.7%.